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Corollary. If $a=b=c$, $N=\frac{5}{8}a^4$, $D=a^3(1+\log 3)$.

$$\therefore M=\frac{5a}{8(1+\log 3)}. \quad M_1=\tfrac{1}{3}a(1+\log 3).$$

CALCULUS.

207. Proposed by F. P. MATZ, Sc. D., Ph. D.

If K represents the complete elliptic integral of the first kind, prove that

$$\int_0^1 \frac{K d\kappa}{1+\kappa}=\tfrac{1}{4}\pi^2.$$

Solution by S. A. COREY, Hiteman, Iowa.

As $K=\int_0^1 \frac{dx}{\sqrt{[(1-x^2)(1-\kappa^2x^2)]}}$, the definite integral may be written

$$\int_0^1\int_0^1 \frac{dx\,d\kappa}{(1+\kappa)\sqrt{[(1-x^2)(1-\kappa^2x^2)]}},$$

$$\text{or} \int_0^1 \frac{1}{(1-x^2)^{\frac{1}{2}}}\left[\int_0^1 \frac{d\kappa}{(1+\kappa)(1-\kappa^2x^2)^{\frac{1}{2}}}\right]dx \text{..........} (1).$$

If $r=\kappa+1$, $\int_0^1 \frac{d\kappa}{(1+\kappa)\sqrt{(1-\kappa^2x^2)}}=\int_1^2 \frac{dr}{r\sqrt{X}}=$

$$-\left[\frac{1}{\sqrt{a}}\log\left(\frac{\sqrt{X}+\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right)\right]_1^2=-\frac{1}{\sqrt{a}}\log\left(\frac{1}{1+\sqrt{a}}\right)=\frac{1}{\sqrt{a}}\log(1+\sqrt{a}),$$

in which $a=(1+x^2)$, $b=2x^2$, and $X=(1-x^2)+2x^2r-x^2r^2$.

The definite integral (1), after substituting, becomes,

$$\int_0^1 \frac{\log[1+\sqrt{(1-x^2)}]}{(1-x^2)}dx, \text{ or, if } x=\frac{1-z^2}{1+z^2}, \quad \int_0^1 \frac{1}{z}.\log\left(\frac{(1+z)^2}{1+z^2}\right)dz$$

$$=2\int_0^1 \frac{\log(1+z)}{z}dz-\int_0^1 \frac{\log(1+z^2)}{z}dz=2.\frac{\pi^2}{6}-\frac{\pi^2}{2.6}=\frac{\pi^4}{4}.$$

208. Proposed by F. P. MATZ, Sc. D., Ph. D.

Solve the differential equation

$$(a^2+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0.$$

Solution by A. H. HOLMES, Brunswick, Me.

$$(a^2+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0=\frac{d}{dx}\left[(a^2+x^2)\frac{dy}{dx}\right].$$